Technical Comments

Comments on "Role of Aluminum in Suppressing Instability in Solid Propellant Rocket Motors"

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Introduction

A RECENT paper[1] seeks to explain the mechanism by which the powdered aluminum ingredient in solid rocket propellants suppresses oscillatory combustion. The paper also purports to summarize the historical background of this subject. This paper is sufficiently misleading to motivate some corrections.

The part of Ref. 1 that is original consists of an extension of an earlier paper by the same authors,[1] an extension of an analytical model for perturbation behavior of solid propellant combustion. In the new work, it is found that the thermal inertia of the accumulated aluminum on the burning surface tends to reduce the response of the combustion to incident pressure disturbances. The analysis in Ref. 1 appears to be relevant and correct, and is a useful contribution. The authors do not note that qualitatively similar results can be inferred from two earlier analyses[2,3] by assuming endothermic surface reactions, or that the analytical result of Ref. 1 can be extracted as a special case from another earlier analysis.[4]

It is the purpose of this Comment to point out that none of these analyses (including Ref. 1) deals with the combustion of the aluminum ingredient of the propellant or the effect of the resulting condensed phase reaction products, subjects that are critically related to the topic in the title of Ref. 1, but not obviously amenable to this type of analysis. In this Comment, a brief summary is made of the present knowledge of aluminum combustion as it relates to the title topic of Ref. 1, which will help to clarify the aspects of the "Role of Aluminum in Suppressing Instability in Solid Propellant Rocket Motors" that are neglected in Ref. 1.

Reference 1 also presents introductory and historical material, presumably to provide experimental justification for the analytical model and support for conclusions drawn from the analysis. An effort will be made to clarify some misleading or erroneous aspects of this introductory and historical material, and to provide references conspicuously absent in Ref. 1. It will become clear that the mechanism proposed in Ref. 1 is not sufficient to explain the role of aluminum in suppressing instability, and that other processes such as metal combustion and particulate damping are usually equally or more important (even under the conditions considered in Ref. 1). These points will be supported with references available prior to Ref. 1, although they may be supplemented with more recent references in some instances when the arguments are made more decisive by such referencing.

Aluminum Behavior

The detailed behavior of aluminum in the propellant combustion zone is as follows. Because of its high boiling point, aluminum tends to accumulate on the burning surface in a condensed phase,[5–9] protected from ignition by a thin layer of aluminum oxide.[7,9,10] The aluminum leaves the burning surface primarily as agglomerates in the 30–300 μ diam range (details depending on propellant variables). The accumulation process on the surface evidently involves sintering of individual particles together,[7,9] probably concurrently with attainment of the melting point of the metal. This is followed eventually by complete breakdown of the oxide "skin" on original particles and formation of reacting agglomerate droplets.[5–9] The extent of reaction on the burning surface is unknown and probably dependent on ingredient and formulation variables. Ignition is often seen to occur concurrently with release from the surface, but with some propellants the luminous metal droplets remain for some time on the surface, and with other propellants no droplet ignition occurs until after separation from the surface.[11]

Regardless of where ignition of the aluminum occurs, most of its combustion occurs after it leaves the propellant surface, and proceeds for a considerable distance from the surface. The burning droplet is seen in photographic studies[5–7,11] to have one or more "caps" of molten oxide, which presumably grow during droplet burning and leave a "residue" droplet upon burnout of the aluminum. In addition, the droplet is surrounded by a detached reaction zone in which smaller Al_2O_3 droplets are formed (mostly in the 0–2 μ diameter range).[5–7,11,12] Quantitative data on the amount of oxide in the "smoke" form and the amount in the "residue" form are unavailable, as are the particle size distributions. However, the distribution is necessarily bimodal due to the two modes of Al_2O_3 formation, with one peak in the vicinity of 1 μ and another of comparable magnitude in the 5–20 μ range, the latter being dependent on original aluminum particle size and degree of agglomeration.[7,9,13]

Perturbation Behavior

The perturbation behavior of the combustion and the flow-combustion interactions is much less well understood than the steady-state behavior. However it seems reasonable to expect that oscillatory combustion of the aluminum will occur through interaction of flow oscillations with the accumulation-agglomeration-ignition-combustion of the metal, and there is substantial evidence to support this.[7,14] Reference 1 considers only the thermal inertia of the aluminum, not its combustion. Thus it seems unlikely that the analysis presented there would fully elucidate the "Role of Aluminum in Suppressing Instability in Solid Propellant Rocket Motors." The supplementary qualitative discussion in Ref. 1 makes no mention of combustion of the aluminum, or of references in the literature on its possible importance to perturbation behavior.

The interaction of the gas oscillations with the Al_2O_3 droplets has been treated analytically in the literature in a fairly realistic manner,[15,16] and the validity of the analysis relative to propellant applications has been supported experimentally.[15,17–19] The dependence of attenuation on frequency and droplet size given by the analysis is shown in Fig. 1 for a typical set of propellant parameters. As indicated in Ref. 1, the attenuation effect is very high at high frequency, especially for the 1 μ fraction of the product droplets. Likewise the attenuation by 1 μ droplets is low below 2000 cps. However this situation can be very misleading, as the amplification rate due to combustion varies with frequency in a rather similar way over the frequency range 500–5000 cps. Further, it is clear from the figure that one should be concerned with the coarser fraction of the Al_2O_3 droplet population when considering the

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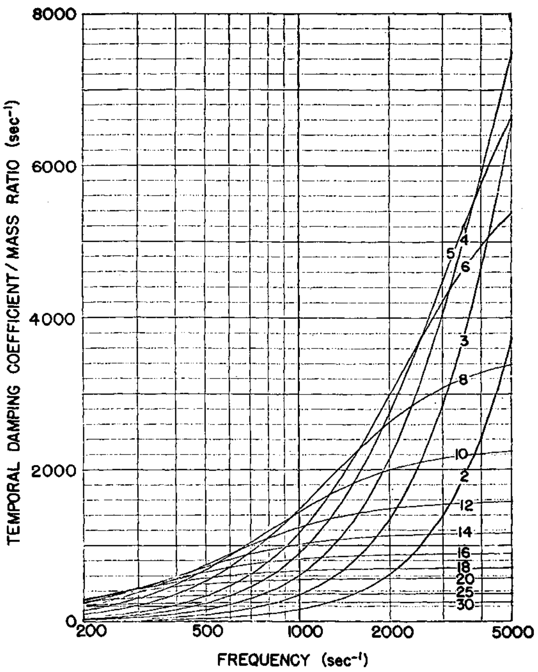

Fig. 1 Particulate damping vs oscillation frequency for various particle sizes (indicated by diameter in micrometers). The ordinate is the damping, divided by the ratio (mass of condensed phase/mass of gas phase) in propellant reaction products. Graph applies for mass ratios in the range 0.1 to 0.8, with properties corresponding to Al_2O_3 droplets in propellant reaction products at 300 psia.

500–2000 cps range. This fraction can produce much more damping in this frequency range than can the fine fraction of Al_2O_3, and the effect is relatively insensitive to the nature of the droplet material. This high attenuation and insensitivity to material is in contrast with observations made in Ref. 1 which states "...experimental attempts to stabilize a rocket motor by adding not metallic aluminum but simply fine aluminum oxide to the propellant have met with only scant or doubtful success." Reference 1 gives no references to support this position, and a contrary view was subscribed to earlier[20] by one of its authors. The earlier report[22] says "aluminum oxide is almost as good as aluminum... [and] a number of materials, including very fine silicon dioxide, have been effective also."

Ultimately, the importance of different factors contributing to stability must be established in quantitative terms, as functions of relevant variables involving amplification and damping of oscillations. This is illustrated here by a very simple calculation. Consider a propellant charge burning only on a right circular cylindrical interior surface, with gas oscillations in the first axial mode. The growth rate of pressure oscillations (in the absence of damping) is given approximately† by the expression[8]

$$\alpha_c = 4a\rho_p(S_p/A_c)(\bar{r}/\bar{p})(\mu/\epsilon)f$$

Noting that the frequency is given by $f = a/2L$, and considering combustors of different size with constant $L/D = 5$, the "combustion Alpha" is

$$\alpha_c = 0.616\ (\mu/\epsilon)f$$

where a = velocity of sound = 3500 fps, ρ_p is propellant density = 0.0022 slugs/in.^{3}, $\bar{r}$ = burning rate = 0.5 in./sec, $\bar{p}$ = pressure = 500 lb/in.^{2}, S_p = burning surface area = πDL, A_c = cross-sectional area of the cylinder = $\pi D^2/4$, f = frequency, and μ/ϵ is the pressure coupled mass burning rate response function. It can be shown from the particle attenuation theory[15,16] that the contribution to the oscillations by the Al_2O_3 droplets is given by

$$\alpha_{d\ \max} = 0.586\,f$$

where c_m, the ratio of mass of the droplets to mass of the gas products, was taken to be 0.39. This relation applies when the droplet size is optimum for the frequency in question—i.e., the upper envelope of the curves in Fig. 1.

Curves of α_c and α_d vs frequency are shown in Fig. 2 for conditions of parameters typical of aluminized propellants. The example shows that, given the optimum particle size, the particulate damping alone would almost be sufficient to assure stability at any given frequency. In practice, of course, one has a specific particle size distribution resulting from the aluminum combustion, a situation that would give less damping at every frequency than indicated by the solid α_d curve. The damping due to a specific bimodal droplet size distribution is shown by the broken curves in Fig. 2. It can be seen there that the coarse residue fraction provides about 30% of the damping necessary for stability in the frequency range 0–3000 cps, with the fine (1 μ) fraction beginning to be more important somewhere above 5000 cps. Thus the particle attenuation cannot be dismissed as proposed in the "Role of Aluminum in Suppressing Instability in Solid Propellant Rocket Motors," and other materials of comparable "durability" and particle size would be expected to also provide substantial attenuation.

History of the Combustion Instability Problem

One section of 1000 words in Ref. 1 is about history. The brevity necessarily leaves the result superficial. Unfortunately it is also substantially inaccurate. Since the unclassified aspects of the history have been reported previously[22–26], comments here will be restricted to a few corrections and clarifications of Ref. 1.

1) In the early 40's unstable burning was observed in many rocket motors using the then-available low-frequency response instrumentation. The associated oscillatory behavior is now

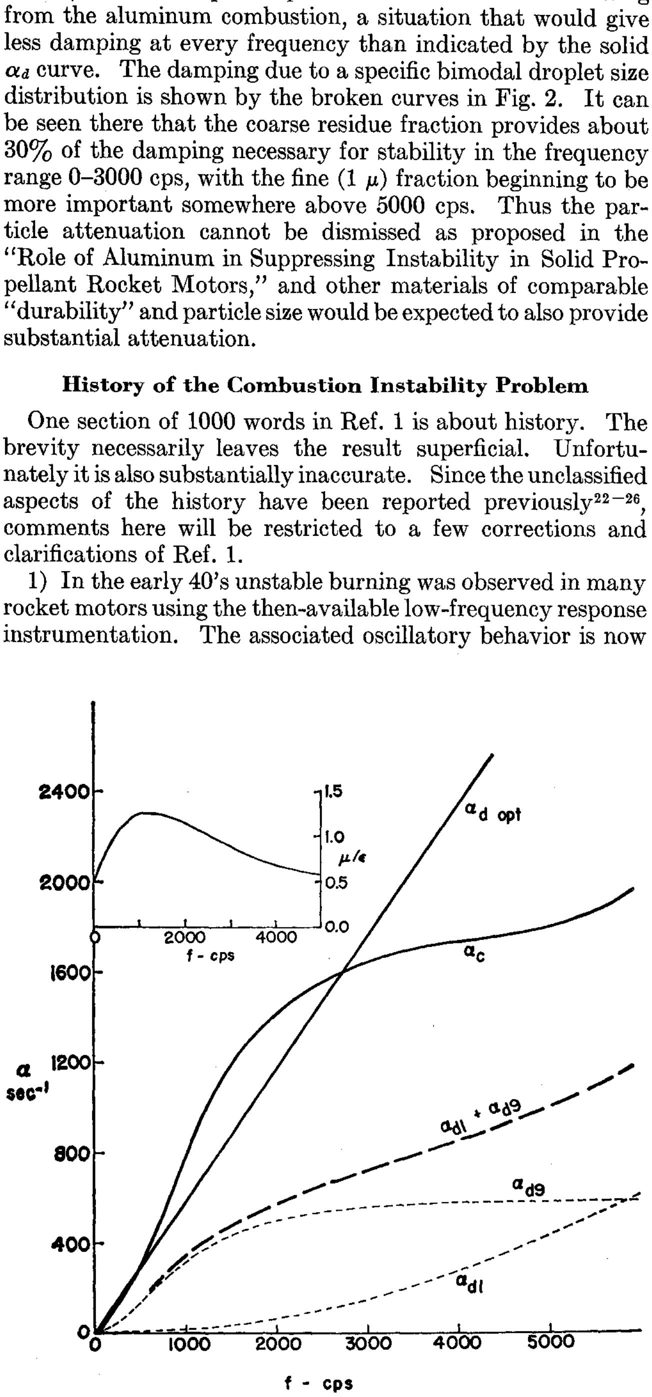

Fig. 2 Comparison of combustion amplification (α_c) and particulate damping (α_d). Numerical designation on α_d indicates droplet diameter, in micrometers. Based on a propellant with 18% aluminum.

† Certain recognized partially compensating effects involving distribution of burning area, presence of mean flow, and nonisentropic effects are neglected for simplicity.

known to have been in the frequency range 2000–50,000 cps range, but was not observed until high-frequency response instrumentation was used starting in 1947.[21,27,28]

2) It is noted in Ref. 1 that oscillatory combustion was encountered in 1946 when potassium perchlorate was replaced by ammonium perchlorate in composite propellants. The authors explain that this experience can be rationalized by their theory. They fail to note that their theory has been interpreted for the frequency range up to 2000 cps, while the oscillatory combustion in 1946 was at much higher frequency (e.g., Ref. 20–24). The same comment pertains to the authors' explanation of oscillatory combustion with double base propellants. What is more significant, the authors fail to note that there is no shortage of analytical models for explaining combustion instability, mostly preceding theirs. The problem is to find models that are realistic enough to have predictive value instead of adjustable parameters and hindsight.

3) Reference 1 describes in somewhat elusive terms the problems with combustion instability in the large motor programs prior to 1956. The reference seems to say that unaluminized propellants were in use, that mechanical damping devices were considered or tried and found to be "impractical," and that introduction of aluminum in the propellant led to stable behavior. No references are provided relative to these points, and this writer could find none that supported them. The fact is, the large motor programs mentioned in Ref. 1 did not exist prior to 1956, published no reports on combustion instability problems when the programs did exist, used only aluminized propellants, and reported no consideration of mechanical damping devices.

4) Reference 1 makes frequent mention of the effectiveness of aluminum in suppressing instability, using such terms as "remarkable potency," "remarkable effect," and "unique in its effectiveness." It also argues at various points that, although particulate damping may be the reason for the effectiveness at high frequency, the thermal inertia effect is responsible for stabilization in the frequency range below 2000 cps. The historical fact is that aluminum is indeed consistently (but not uniquely) effective at high-frequency, but has not been demonstrated to be consistently effective at frequencies below 2000 cps. Even though reports on experience in development programs are usually classified, there are several unclassified references (not mentioned in Ref. 1) to systematic studies of the effect of aluminum on combustion instability in this "lower" frequency range,[23,25,29–32] and these references certainly contradict any claim of remarkable potency for suppressing instability. In fact a case can be made that combustion of aluminum caused the instability,[7,8,14,25,30] which occurs only rarely in rocket motors in this frequency range with unaluminized propellants.

In summary, Ref. 1 presents an extension of an earlier analysis to account for the effect of the melting and thermal inertia of accumulated aluminum on the burning surface. The calculation probably has at least qualitative validity for this purpose, but represents only one, possibly secondary, aspect of the effect of aluminum. The position that this is the "role of aluminum in suppressing instability in solid propellant rocket motors" (the title of Ref. 1) is not supported by the facts, and the arguments in Ref. 1 are "supported" by erroneous information and neglect of contradictory evidence.

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Comments on "Study of Nonlinear Systems"

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A FEW points of the recent comments by Rao,[1] have been advanced in the light of several published materials.

It is a new contribution to apply the direct interchange between dependent and independent variables. Nonlinear, ordinary differential equations, especially Riccati equations, have been actively discussed among radio engineers for nonuniform transmission lines where the line parameters vary continuously along the line length. However, a preliminary literature search seems to exclude the direct interchange between dependent and independent variables.

One of the key points in obtaining closed form solutions by factorizations in Ref. 1 has been the following relationship:

$$(d/dx)[F(x)/f(x)] = \lambda f(x) \tag{1}$$

This is a Bernoulli nonlinear differential equation for $f(x)$ when $F(x)$ and λ are specified,

$$df(x)/dx = f(x)\ (d/dx)[\ln F(x)] - [\lambda/F(x)]f^3(x) \tag{2}$$

This interrelationship between $f(x)$ and $F(x)$ is restricting a possible wider application of Rao's method. In the process of generalizing the original Konyukov's nonlinear differential equation[2,3] a similar relation has been derived for variable coefficients.[4] It is also noted that the book by Murphy[5] goes into a detailed discussion on Abel's nonlinear differential equations, including the separable case of Eq. (1).

At this point, Rao's method has been used for a generalized Konyukov's nonlinear differential equation. The direct interchange of dependent and independent variables has been given to

$$xd^2x/dt^2 + P(t)(dx/dt)^2 + Q(t)x^3 + R(t)x^2 = 0 \tag{3}$$

where $P(t)$, $Q(t)$, and $R(t)$ are arbitrary functions of the independent variable t. Then, the result is

$$d^2t/dx^2 = [P(t)/x]\,dt/dx + [Q(t)x^2 + R(t)x](dt/dx)^3 \tag{4}$$

Evidently this appears not solvable for general $P(t)$, $Q(t)$, and $R(t)$. In contrast, the published materials show how to obtain closed forms for the following cases:

$P(t) = -2,\quad Q(t) = \text{constant},\quad R(t) = \text{constant},$ Ref. 2

$P(t) = -2,\quad Q(t) = \text{arbitrary},\quad R(t) = \text{constant},$ Ref. 3

$P(t) = -2,\quad Q(t) = \text{arbitrary},\quad R(t) =$ a solution of Bernoulli's equation Ref. 4

Equation (4) is solvable if $P(t)$, $Q(t)$, and $R(t)$ are all constant, since it then becomes a Bernoulli's nonlinear differential equation for dt/dx. Thus, it appears that another key point of why Rao's method works successfully on Eq. (2) of Ref. 1 is that it contains no variable coefficients of t.

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Comment on "Unsymmetrical Bending of Shells of Revolution"

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IN Ref. 1, Blech has presented a set of eight first-order ordinary differential equations for the bending analysis of unsymmetrically loaded shells of revolution on the basis of Sander's first-order shell theory. This formulation was first suggested by Goldberg et al.[2] and has been successfully employed by several authors for the equilibrium, stability and vibration problems of shells.

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